2.2-2

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May 13, 2015

SELECTION-SORT(A)	**Cost**	**Time**
for $j = 1$ **to** $A.length - 1$	c_1	n
$\quad min = j$	c_2	$n - 1$
$\quad$ **for** $i = j + 1$ **to** $A.length$	c_3	$\sum_{j=1}^{n-1} t_j$
$\quad\quad$ **if** $A[i] < A[min]$	c_4	$\sum_{j=1}^{n-1}(t_j - 1)$
$\quad\quad\quad min = i$	c_5	0 **to** $\sum_{j=1}^{n-1}(t_j - 1)$
$\quad$ swap($A[min], A[j]$)	c_6	$n - 1$

We will now calculate the running time, $T(n)$, of SELECTION-SORT:

$$\begin{aligned}
T(n) &= c_1 n + c_2(n-1) + c_3 \sum_{j=1}^{n-1} t_j + c_4 \sum_{j=1}^{n-1}(t_j - 1) + kc_5 \sum_{j=1}^{n-1}(t_j - 1) + c_6(n-1), \\
&= c_1 n + (c_2 + c_6)(n-1) + c_3 \sum_{j=1}^{n-1} t_j + (c_4 + kc_5) \sum_{j=1}^{n-1}(t_j - 1), \\
&= c_1 n + (c_2 + c_6)(n-1) + (c_3 + c_4 + kc_5) \sum_{j=1}^{n-1} t_j - (c_4 + kc_5)(n-1), \\
&= c_1 n + (c_2 + c_6 - c_4 - kc_5)(n-1) + (c_3 + c_4 + kc_5) \sum_{j=1}^{n-1} t_j. \qquad (1)
\end{aligned}$$

1 Best case running time

In the **BEST CASE** running time, the list of input will already be sorted. Thus, the body of **if** is never step in, and $k = 0$. we obtain that $t_j = j + 1$, for every choice of j. Thus,

$$\sum_{j=1}^{n-1} t_j = \frac{1}{2}n(n+1) - 1 = (\frac{n}{2} + 1)(n - 1)$$

Substituting this into the last term of Eqn. (1) yields,

$$T(n) = c_1 n + (c_2 + c_6 - c_4)(n-1) + (c_3 + c_4)(\frac{n}{2} + 1)(n-1) \tag{2}$$
$$= \frac{c_3 + c_4}{2} n^2 + (c_1 + c_2 + \frac{c_3}{2} - \frac{c_4}{2} + c_6)n - (c_2 + c_3 + c_6) \tag{3}$$

which can be simplified to the linear equation $T(n) = An^2 + Bn + C$ where

$$A = \frac{c_3 + c_4}{2} > 0,$$
$$B = c_1 + c_2 + \frac{c_3}{2} - \frac{c_4}{2} + c_6, \quad \text{and,}$$
$$C = -c_2 - c_3 - c_6 < 0.$$

Therefore, the **BEST CASE** running time of the SELECTION-SORT Algorithm equals:

$$\boldsymbol{T(n) = An^2 + Bn + C}$$

2 Worst case running time

We will now look at the **WORST CASE** for SELECTION-SORT:

- In the worst case, the **if** statement is invoked on every occasion.
- This means $k = 1$

Substituting t_j with j into the last summation in Eqn. (1) yields,

$$\sum_{j=1}^{n-1} t_j = \frac{1}{2} n(n+1) - 1 = (\frac{n}{2} + 1)(n-1)$$

Thus, Eqn. (1) becomes,

$$T(n) = c_1 n + (c_2 + c_6 - c_4 - c_5)(n-1) + (c_3 + c_4 + c_5)(\frac{n}{2} + 1)(n-1),$$
$$= \frac{c_3 + c_4 + c_5}{2} n^2 + (c_1 + c_2 + \frac{c_3}{2} - \frac{c_4}{2} - \frac{c_5}{2} + c_6)n - (c_2 + c_3 + c_6)$$

a *quadratic function* of n, the input sequence length, where,

$$A' = \frac{c_3 + c_4 + c_5}{2} > 0,$$
$$B' = c_1 + c_2 + \frac{c_3}{2} - \frac{c_4}{2} - \frac{c_5}{2} + c_6, \quad \text{and,}$$
$$C' = -c_2 - c_3 - c_6 < 0.$$

Therefore, the **WORST CASE** running time of the SELECTION-SORT Algorithm also equals:

$$\boldsymbol{T(n) = An^2 + Bn + C}$$